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13. ABSTRACT *(Maximum 200 words)*

Our research efforts under the ARO Grant have developed a robust control design methodology for distributed interactive systems, such as they arise within the technology of smart materials and structures. The proposed approach is based on the Partial Differential Equations (PDE) that model the structures from first physical principles. As such, this methodology covers the entire range of frequencies, and, moreover, accounts for new pathological phenomena, of which there is no counterpart in the case of lumped (finite dimensional) systems. A benchmark problem of paramount importance in itself, which also serves as a vehicle to test the proposed PDE-based approach, is the noise reduction (or structural acoustic) problem. Here the goal is to reduce, or attenuate, or dampen out the unwanted noise field, which is caused within an acoustic chamber by an

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external source. To this end, one makes use of the destructive interference of the acoustic pressure generated by one of its moving elastic walls, under the bending action produced by wired smart materials bonded to it. A balanced combination of passive (stabilization) and active (optimization) controls was used, which turned out to be effective over a wide band of frequencies, high and low frequencies. When tested in a simplified canonical model, the PI's design produced a 70% noise reduction rate. Numerical tests are available. This research has resulted in a large number of publications in top-ranked professional journals by the PI's and their Ph.D. students, which are listed in the full size Final Technical Report. Many results of this research were presented by the PI's at numerous international conferences in US, Europe and China.

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FINAL TECHNICAL REPORT

Summary of research accomplishments under the ARO Grant DAAH04-96-1-0059 entitled: ***"A partial differential equation approach to robust control design of smart materials and structures: theoretical and computational aspects"***

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Grant Period: April 1, 1996–September 30, 1999

1 Orientation

The goal of the present research project, expired as of September 30, 1999, is the development of a Partial Differential Equation-based robust control design approach to physically significant distributed parameter systems (DPS), by the use of *smart materials and structures.*

To this end, general control and optimization DPS strategies and methodologies, developed by the PI's over the years, are brought to bear on intelligent structure models, which—to be sure—introduce a whole array of new problems and additional difficulties.

It is a felicitious fact that robust control problems for DPS, with boundary/point control, such as they are induced by smart material technology, fit 'perfectly' into the general mathematical apparatus of PDE's and infinite-dimensional control theory with unbounded control/observation operators, which was developed by the PI's over the past two decades. An account thereof is reported, in part, in the three-volume Cambridge University treatise [2] and in the companion Kluwer's volume [67] by the PI's.

Even in the case of optimal control theory for single classes of PDE's - parabolic and hyperbolic PDE's - with boundary/point control and corresponding Riccati equation [2], new phenomena arise, of which there is no counterpart in the finite dimensional theory. Accordingly, they need to be accounted for, to devise a robust control design.

A benchmark problem of paramount importance in itself, which also serves as a vehicle to test the proposed PDE-based methodology is the *structural acoustic problem.* Here, the goal is to reduce, or attenuate, or dampen out the unwanted noise field caused within an acoustic chamber by an external source, by means of the destructive interference of the acoustic pressure generated by one of its moving elastic walls, under the bending action produced by wired smart materials bonded to it. To achieve this goal, the PI's PDE-based approach consists of a balanced combination of *passive* and *active* controls, which turn out to be effective over a wide band of frequencies, *high* as well as *low* frequencies. This is so since the PDE-based methods employed in the PI's research have the intrinsic feature of accounting for the entire range of frequencies. Thus, they intrinsically avoid the threat of stability being deterioriated or even destroyed by a potential high frequencies spillover, which—by contrast—is always present in other approaches to DPS, such as those that replace them by lumped systems at the outset.

As to the **results** achieved under our approach, we are pleased to report that the theoretical and numerical PDE-methodology pursued by the PI's and their past and present Ph.D. students and based on optimal control techniques, when tested on a simplified canonical structural acoustic problem, show a *reduction rate of the unwanted noise in the acoustic chamber by approximately* 70% (Appendix A).

In the present Final Technical Report we briefly report, by topics, the main research accomplishments achieved under the present ARO Grant DAAH04-96-1-0059 by the PI's and their past and present Ph.D. students. In short, as already pointed out above, in a broad sense, the research reported here has revolved

around one main theme: *the noise reduction problem.* In synthesis, the model of the structural acoustic chamber couples a wave equation for the pressure in the interior of the chamber with an elastic equation for the displacement of its elastic wall. These two PDE equations are strongly coupled, with coupling taking place in the boundary conditions of each. Thus, the independent study of each single component separately—the *wave equation* on one hand, and the *elastic equation* in any of its several realistic forms on the other, possibly subject to thermal effects as well—is duly emphasized and forms an intrinsic part of the present project.

2 Topics

(i) References of group A. Work on optimization and Riccati equations. The PI's have now completed the first two volumes (1,100 pp.) of an in-depth, up-dated comprehensive treatise on optimal control theory for Partial Diferential Equations, which is an outgrowth of their 1992 Springer Verlag Lecture Notes. Both continuous theory and approximation theory are included. Emphasis is on boundary control or point control action, such as it arises in the context of smart material and smart structures models. This treatise is almost entirely based on the original research work by the PI's and their co-authors on both control problems and PDE-problems. A third volume, almost completed, is centered on additional aspects of hyperbolic-like optimal control problems, such as their delicate numerical approximation theory, which requires a very special regularization approach (see Section 2.3). The noise reduction problem will be included as well. This shows that the PDE's-based approach proposed and followed by the PI's is instrumental in capturing new pathological phenomena, of which there is no counterpart in finite dimensional theory.

(ii) References of group B. The noise reduction problem. Simplified canonical models. The work conducted under this project contains 2-D and 3-D canonical models under the following simplified assumptions: (i) the moving wall is purely *elastic* (no thermal effects), (ii) is *flat*, and (iii) the differential operators of the model have *constant coefficients.* A main thrust of the present proposal is to eliminate all of the above restrictions and consider moving walls which by contrast are *curved* (shells); which account for *thermal* effects, and where the differential operators have *variable coefficients.* Both parabolic/hyperbolic and hyperbolic/hyperbolic models of the acoustic chamber were studied.

(iii) References of group C. Thermo-elastic systems. Thermal effects, in particular thermal dissipation, play an important role in inducing stability properties in elastic systems. This is the case, in particular, for the elastic moving wall of an acoustic chamber. On the other hand, the effects of thermoelasticity can be detrimental to exact controllability properties [Avalos-Lasiecka, Sicon]. Thus, the PI's have studied dynamical properties of thermo-elastic systems per se, both as a topic of interest in itself under the present ARO project, as well as a preliminary step toward their inclusion into a more realistic description of the acoustic chamber. The role of the rotational inertia term was characterized: if present, it induces an hyperbolic-dominated dynamics; if absent, it produces an analytic dynamics. Nonlinear models were included as well [26].

(iv) References of group D. Non-linear elastic systems and shells. As a preliminary step toward the study of fully non-linear structural acoustic models with curved elastic walls, the PI's have investigated non linear systems of dynamic elasticity (modeling large displacements) as well as certain shell equations described on curved domains, with special symmetries. As to non-linear elastic systems, both modified von Karman systems as well as full von Karman systems were included. Outstanding open

problems such as the uniqueness of finite energy solutions were solved in the affirmative [41, 42]. The PI's work includes studies on non-linear *shells* with special geometries [16, 48, 54, 59, 60, 61].

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